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Statistical Characterization of Acceleration Levels of Random Vibrations during Transport

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Random vibration tests are an efficient way to simulate the mechanical vibratory effects caused by transportation. The usual method is only concerned with the frequency distribution pattern of the signal using the average power spectral density. This work offers an additional method based on detailed analysis of instantaneous acceleration levels of a real road transport, which enables modelling of the statistical distribution of these levels. Continuous recording of acceleration signal all along the journey permits confirmation that this statistical distribution is not a Gaussian distribution but a modified Gaussian distribution, for which parameters are estimated and discussed. Therefore, it is possible to evaluate the transport severity by working out the appearance probability of acceleration levels greater than a fixed threshold and also the statistical moments, i.e. second order moment which gives the root mean square value together with fourth order moment (kurtosis) which evaluates the difference between the experimental distribution and the Gaussian distribution. Copyright © 2011 John Wiley & Sons, Ltd.

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INTRODUCTION

Dynamic stresses during transport (vibrations or shocks) are the principal causes of damage to cargo. This is why the packaging qualification requires a series of tests, particularly vibration tests, which can be carried out in the real transport environment or in the laboratory. Making the most realistic possible transport simulation in the laboratory is an essential concern for the industry. Indeed, it is more efficient and less expensive to assess the performance of a packaging unit in the laboratory than during a real transport. On the other hand, the random nature of transport vibrations means that it is not possible to carry out statistically reliable testing *in situ*; therefore, to create a quality procedure, more than one journey needs to be considered.

Random vibration tests are the most efficient way to reproduce the mechanical vibratory effects of transport. In this method, a sample is placed in a vibrating system and subjected to a random vibratory force determined by an average power spectral density (PSD).[1] The random vibration simulation can be created either from a standard PSD provided for specific transport types, or from real transport vibration recordings.

This method considers the frequency distribution of transport vibrations,[2,3] with severity being determined by the root mean square (RMS) value of the acceleration signal. Therefore, this method, although widespread, ignores an essential part of the information conveyed by the signal, i.e. the

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distribution pattern of acceleration levels. If we measure this acceleration signal level during a lab test, we will confirm its Gaussian nature; although it has been proved that the level distribution for the acceleration signal of a real road transport is not.[4,5] This must be taken into account, as we know that exceptional events with highly intensive levels have a serious impact on transport severity.

Previous research has analysed and synthesized this non-Gaussian characteristic of road vehicle vibration signals. Rouillard[6] proposed a modified Rayleigh distribution, which is built on the instantaneous magnitude of the acceleration signal obtained from the Hilbert transformation called *vibration intensity*. In a similar work, Martinez[7] proposed a modified Weibull distribution model built on the moving RMS value of the vibration signal, which depended on the window width and the incremental step for computing the RMS time history. This model actually has a relatively large number of parameters, which leads to several possible combinations of solutions. This can be corrected by evaluating errors between the fitted and measured values of five statistical parameters: mean, median, standard deviation, skewness and kurtosis. Finally, Rouillard[8] proposed a distribution model based on a combination of the Rayleigh and Weibull models, which was shown to offer a characterization of the non-Gaussian vehicle vibrations. All these models are built on the positive part of recorded data obtained from a transformation of the real signal (Hilbert transformation or RMS calculating).

In this article, we will propose a model built on the real instantaneous acceleration signal (positive and negative parts) without any transformation procedure. We will conduct a thorough analysis of the acceleration levels to develop a method leading to the mathematical model of the statistical level distribution. In the long term, this will enable us to find an alternative method for simulating the road vibrations using actual testing systems in the laboratory, knowing that methods are proposed to synthesize such a non-Gaussian signal. As precedent for this work, we can name the work of Smallwood,[9] which converted a Gaussian signal into a non-Gaussian signal with a specific skewness and kurtosis. There is also the method of Rouillard,[10] which consists of a numerical model in conjunction with pre-determined pavement profiles and constant vehicle speed and, finally, the decomposition of vibration signal into constituent Gaussian elements.[8]

METHODOLOGY

The usual method of characterizing transport dynamic stress consists of recording the acceleration signal in determined intervals of time, or named events (Figure 1). The events are recorded according

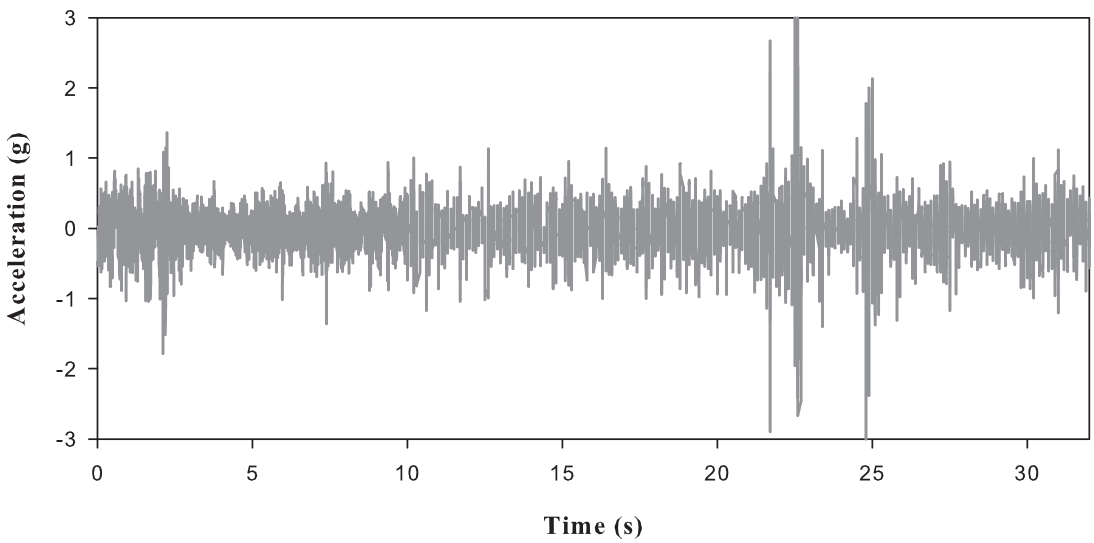

Figure 1. Acceleration signal as a function of time recorded by a self-power accelerometer.

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Table 1. Descriptive statistic of vertical acceleration signal.

	Country road	Motorway
number of measurement points	7 200 000	7 168 000
min, max (g)	–5.06, +5.05	–4.80, +4.10
mean (g)	–3.10–3	4.10–3
standard deviation (g)	0.23	0.22

1 g = 9.81 m/s²

two modes: time or level triggered. In our work to obtain a reference signal, unlike the usual method, we have recorded the entire signal (continuous recording) which enables a statistical study.

The sensor used in this experiment was a self-powered tri-axial piezoelectric accelerometer, with memory capacity of 128 MB. The instrument has been set up for continuous recording in the range of −10 g to +10 g, with a 500 Hz sampling frequency and a low-pass filter at 200 Hz. The total recording time with this configuration is $8^1/_2$ h, which equates to 928 events of 32 s.

To illustrate our analysis method, we have chosen a mode of transport that is used commonly in Europe: a van for normal shipping service, weighing 2.3 T with leaf-spring suspensions. We examined both country road and motorway in France, which are the most representative of French roads. The speed was limited to 90 km/h on country road and 130 km/h on motorway.

RESULTS

The recorded vertical acceleration signal has statistical characteristics, as outlined in the elementary descriptive statistical analysis of Table 1. However, this analysis is not sufficient because the distribution pattern of acceleration levels is ignored. Therefore, we will focus on this aspect. We have developed special software to process the huge quantity of recorded data.

With this software, we are able to define a histogram that shows the probability density of instantaneous values of the signal. Usually for this type of study, histograms with fixed class intervals are chosen. We have chosen the fixed class frequency histogram because it is a better tool for representing the probability density.[11] We chose the number of classes according to Sturges's rule,[12] although this rule has been criticized.[13] The number of classes is given with:

$$k = 1 + \frac{\ln N}{\ln 2}$$

with N being the total number of samples.

Therefore, we have obtained tow histograms for tow trips (Figure 2) with 24 classes of 300 000 samples per class (fixed frequency classes).

It may be noted that the height of classes is, by definition, proportional to the probability density. It is calculated as:

$$h_i = \frac{N_i}{\Delta_i \cdot N}$$

with h_i as the height of class i, N_i as the frequency of class i, which is fixed here, Δ_i as the width of class i and N as the total number of measured samples.

PROBABILITY DENSITY FUNCTION

The statistical distribution of acceleration levels does not seem to be Gaussian according to histograms of Figure 2. In further analysis, we aimed to find the mathematical model of this statistical distribution. We tried different non-linear fits for class centres on the histograms. To get a faster convergence of

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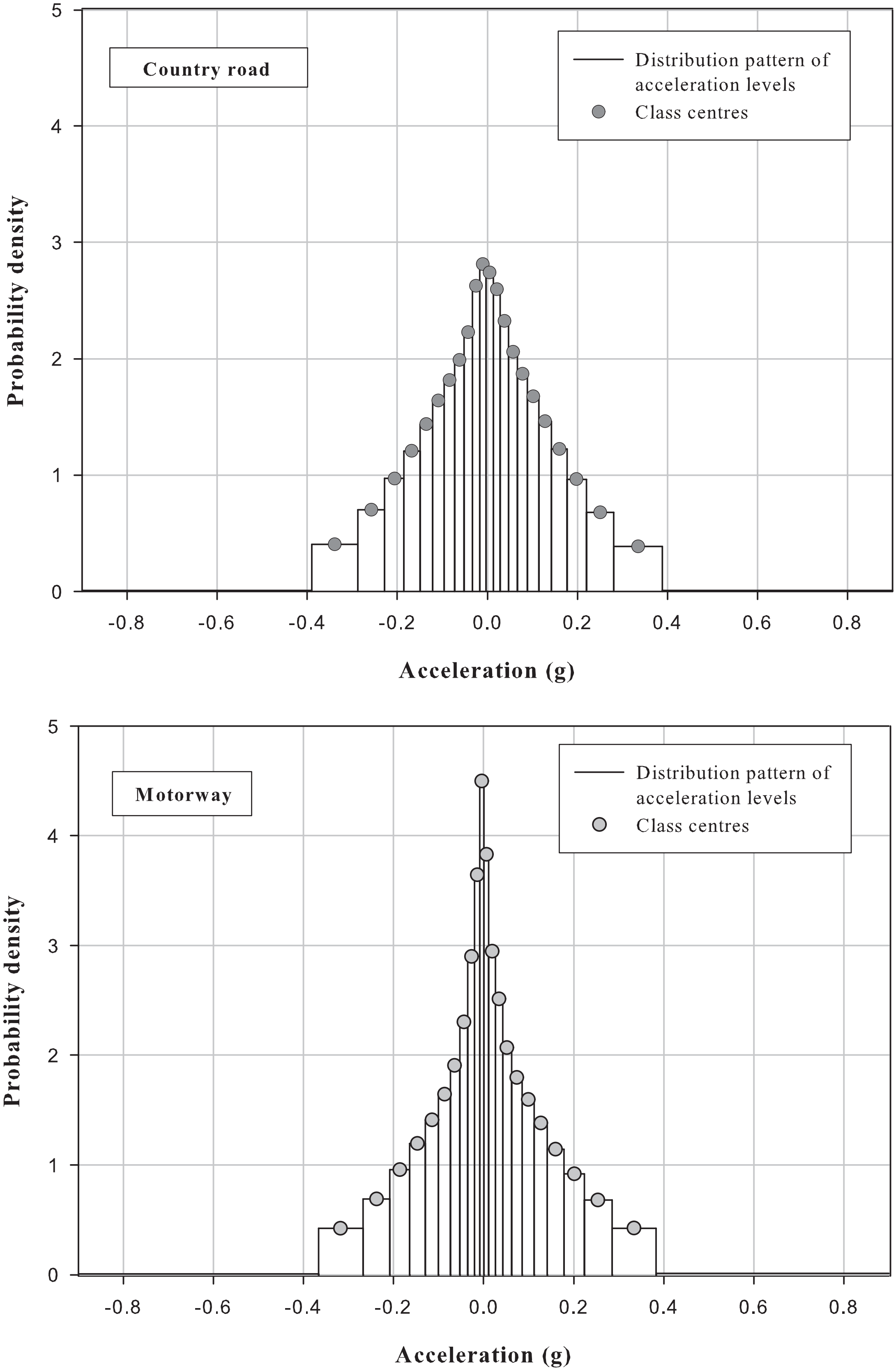

Figure 2. Histograms of the acceleration levels of vertical vibration signal and the class centres.

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Table 2. The best Gaussian and modified Gaussian fit parameters.

	Country road		Motorway	
	Gaussian	Modified Gaussian	Gaussian	Modified Gaussian
a (g^{-1})	2.420	2.812(exp.)	3.107	4.497(exp.)
b (g)	0.142	0.098	0.098	0.030
c	2.000	1.074	2.000	0.641
x_0 (g)	–0.002	–0.002	0.000	–0.003
r^2	0.948	0.996	0.814	0.992

calculating, we used the Marquardt-Levenberg algorithm, which is a modified version of the well-known Gauss-Newton algorithm. The convergence condition is to minimize the sum of squared residuals (the residual being the difference between an observed value and the value provided by the model).

Many equations have been tested to fit class centres (Gaussian, modified Gaussian, pseudo-Voigt, polynomials, fractional, etc.). From all of these equations, we have found that the modified Gaussian was the most relevant with the least number of parameters.

Here is the expression for the modified Gaussian fit:

$$p(x) = a\exp\left[-1/2\left(\frac{|x-x_0|}{b}\right)^c\right]$$

The coefficients of determination r^2 and a, b, c and x_0 parameters in this stage are indicated in Table 2. For comparison, the results of fitting with a classic Gaussian and a modified Gaussian are indicated in Figure 3.

The fit's results, especially r^2 coefficients, show that the modified Gaussian equation fits better than classic Gaussian the class centres. This is true also with all other mathematical models except, of course, for the high degree polynomials. These are dismissed because of the very large number of necessary parameters for their definition.

The classic Gaussian has the same expression with the c coefficient equal to 2. In fact, this exponent parameter enables control of the slope of the distribution tails and the ability to achieve the leptokurtic shape of non-Gaussian distribution with values less than 2. We can observe the influence of this exponent parameter and the others: a and b on the distribution pattern in Figure 4. The a parameter indicates the presence of very low levels and the leptokurtic nature of the distribution, and the b parameter indicates the width of the distribution.

Parameters determined by the Marquardt-Levenberg algorithm lead to the best coefficient of determination, but some modifications are necessary to verify statistical and experimental conditions. It may be noted that if we leave the Marquardt-Levenberg algorithm to determine a parameter, we will find a larger value than the height of the central class of histogram, which can lead to underestimation of transport severity (overestimation of the probability density of very low levels). In fact, the values near x_0 correspond to the noise of the sensor instrument, and the found mean values in Table 1 have no physical signification considering vibratory process so they must be ignored thereafter. Therefore we will consider $x_0 = 0$ and a parameter can be estimated by the height of central class of the histogram. Indeed we will take a = 2.81/g for country road and a = 4.49/g for motorway (with 1 g = 9.81 m/s^2).

To make sure that the analytic fit $p(x)$ could be a probability density function, we have to check this condition:

$$A = \int_{-\infty}^{+\infty} p(x)\,dx = 1$$

Additionally, this model should lead to statistical parameters according to experimental values. So we can make a second condition in order to fix two parameters b and c. this condition will be:

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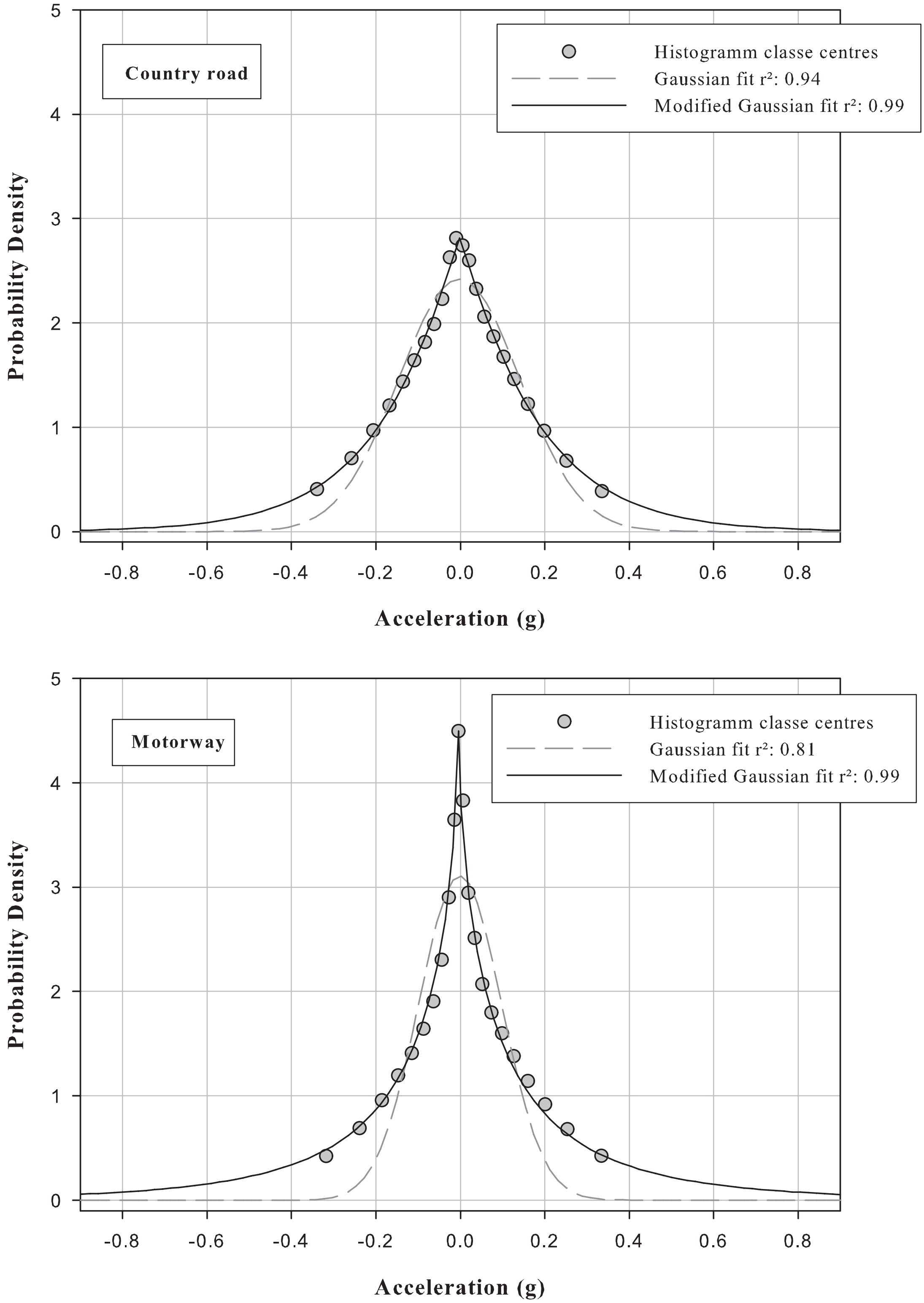

Figure 3. Gaussian and modified Gaussian fits on the histograms of the acceleration level.

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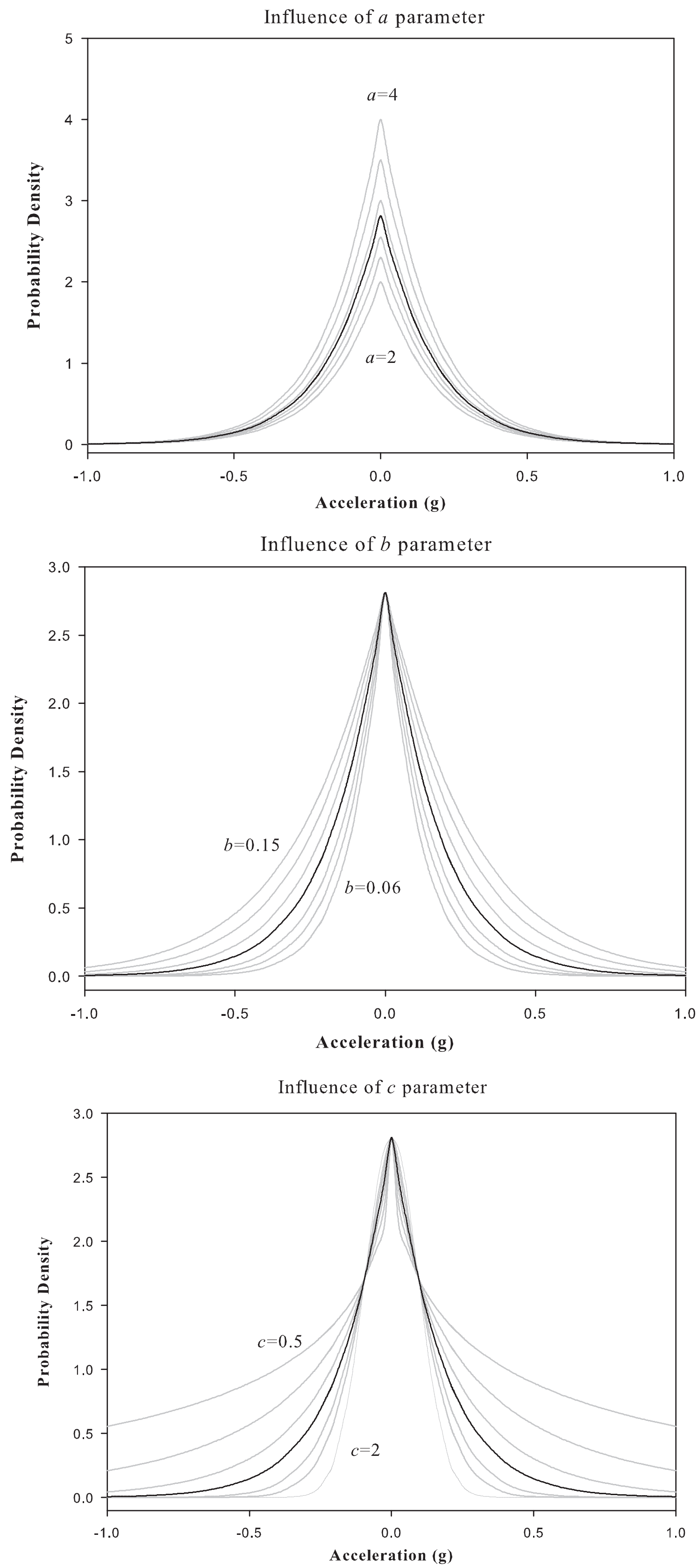

Figure 4. Influence of parameters on the modified Gaussian distribution.

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Table 3. Normalized modified Gaussian model parameters suggested as probability density function of vibration signal generated during transport on two types of road.

	Country road	Motorway
a (g^{-1})	2.812	4.497
b (g)	0.095	0.041
c	1.075	0.799
x_0 (g)	0.000	0.000
r^2	0.996	0.974

$$\sigma^2 = \int_{-\infty}^{+\infty} x^2 p(x)\,dx = \exp.\ value$$

If we consider A and σ^2 as the functions of a, b and c:

$$A = \int_{-\infty}^{+\infty} a \exp\left[-1/2\left(\frac{|x|}{b}\right)^c\right]dx = ab\,2^{1+\frac{1}{c}}\Gamma\left(1+\frac{1}{c}\right)$$

and

$$\sigma^2 = \int_{-\infty}^{+\infty} x^2 a \exp\left[-1/2\left(\frac{|x|}{b}\right)^c\right]dx = ab^3\,2^{1+\frac{3}{c}}\Gamma\left(1+\frac{3}{c}\right)$$

with the conditions below: $b > 0$ and $c > 0$ and the Gamma function $\Gamma(z) = |t^{z-1}e^t dt$ for $z \in C$, $\mathrm{Re}(z) > 0$ *and* $t \in R$

To find an analytic equation for the probability density function that verifies these conditions, we must solve the following equations considering a as constant ($a = 2.81$/g for country road and $a = 4.49$/g for motorway):

$$A(a, b, c) = 1$$

and

$$\sigma^2(a, b, c) = \exp.\ value$$

that is to say:

$$ab\,2^{1+\frac{1}{c}}\Gamma\left(1+\frac{1}{c}\right) = 1$$

and

$$ab^3\,2^{1+\frac{3}{c}}\Gamma\left(1+\frac{3}{c}\right) = \exp.\ value$$

The values found for b and c are shown in Table 3. In fact, with these new parameters the values of r^2 are deteriorated (Table 3) in both of cases, but they are still satisfying.

To complete the statistical analysis of the suggested model, we will calculate the mean (m), the variance (σ^2), the standard deviation (σ), the skewness ($\mu'3$) and the kurtosis (μ'_4). These values are defined from the nth moment, which is:

$$m_n = \int_{-\infty}^{+\infty} x^n p(x)\,dx$$

Table 4. Statistical parameters calculated by mathematical model.

	Country road	Motorway
m (g)	0.000	0.000
σ^2 (g^2)	0.053	0.047
σ (g)	0.230	0.217
μ'_3	0.000	0.000
μ'_4	5.436	8.560

Table 5. Some indicators of transport severity.

	Country road	Motorway
RMS (g)	0.23	0.22
FWHM modified Gauss. (g)	0.26	0.12
FWHM Gaussian (g)	0.54	0.51
S	0.47	0.24

The analytic expression that we find for these moments is:
If n is an uneven number:

$$m_n = 0$$

If n is an even number:

$$m_n = \frac{ab^{n+1}}{c} 2^{1+\frac{n+1}{c}} \Gamma\left(\frac{n+1}{c}\right)$$

Thus $m = m_1$, $\sigma^2 = m_2$, $\sigma = \sqrt{m_2}$ *and* $\mu'_4 = \frac{m_4}{\sigma^4}$.

For our two roads, we have statistical parameters as shown in Table 4. These parameters will give us the information for the level distribution as:

The μ'_3 value is a measure of asymmetry of the probability distribution around the mean value. The calculated value for μ'_3 is equal to zero whatever the trip. It shows that the distribution is completely symmetric.

The μ'_4 value is a measure of peakedness of the probability distribution. For a Gaussian distribution, the value of μ'_4 is equal to 3. A higher value of kurtosis, as for our case, shows the presence of a sharp and high peak and long and fat tails. This means that the probability of both the very low levels and extreme high levels is higher than in a Gaussian distribution.

COMPARISON OF TRANSPORT SEVERITY

To compare the severity of two trips, the most popular indicator is the RMS value of the acceleration signal (RMS). We can calculate this value with the mathematical model of probability density (Table 5).

Indeed:

$$RMS = \sqrt{\int_{-\infty}^{+\infty} x^2 p(x)\,dx}$$

However, the RMS value is a global indicator, which does not take into account the precise level of distribution.

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Our model enables us to analytically calculate the full width at half maximum (FWHM) on the distribution curve of acceleration levels.

$$FWHM_{modified\ Gauss.} = 2b(2\ln 2)^{\frac{1}{c}}$$

We can compare this with a Gaussian distribution with the same value of RMS, which should be:

$$FWHM_{Gaussian} = 2\sigma(2\ln 2)^{\frac{1}{2}}$$

Comparing of the two gives an indicator of the transport severity (S):

$$S = \frac{b}{\sigma}(2\ln 2)^{\frac{1}{c}-\frac{1}{2}}$$

The value of this indicator tends to 1 when the distribution tends to a Gaussian distribution. For a value less than 1 (as in our case), we have a less average dispersion of acceleration signal with the same energy. These comparisons are shown in Table 5.

Another way to compare transport severity could be the comparison of the appearance probability of acceleration levels during the journey. We have estimated the appearance probability of acceleration levels greater than a fixed threshold with the help of the proposed mathematical model for probability density:

$$P[|X| \geq s] = \int_{-\infty}^{-s} p(x)\,dx + \int_{+s}^{+\infty} p(x)\,dx$$

with X: acceleration level and s: acceleration threshold

Moreover, as we have the continuous acceleration signal from throughout the journey, this probability is equivalent to the percentage of the time during which the acceleration level exceeds this threshold. This probability evolution is illustrated as a function of the fixed threshold on Figure 5.

We can note that this probability is greater for country road than motorway for low levels to 0.6 g, and after this threshold the tendency is opposite, with the extreme levels more probable on motorway than on country road.

The association of these indicators, i.e. RMS, S and probability of exceeding the threshold ($P[|X| \geq s]$), gives relevant information to evaluate the severity of transport.

CONCLUSION

We have developed a method in this article that enables us to characterize non-Gaussian random vibrations generated by road vehicles. A statistical model is built on the histogram of acceleration levels obtained from continuous *in situ* recording. The probability density function is determined first by fitting a modified Gaussian function and second by verifying statistical and experimental conditions. The model is summarized to only three parameters. These three parameters are pertinent because each one of them gives information of the level distribution, i.e. the height and the width of the peak and the slope of the distribution tails. A comparison to a Gaussian distribution with the same RMS value is also carried out with the help of FWHM. On the other hand, this method enables us to estimate the appearance probability of acceleration levels and so to compare two different journeys. Therefore, they can lead to the severity indicator in addition to RMS value that is usually used. This method has been illustrated for two specific journeys (French country road and motorway) and has shown satisfying results.

This work is based on the continuous recording of acceleration signals, whereas oftentimes the signal is recorded on a time-triggered basis. Therefore, it might be interesting to study the effect of sub-recording on these results, to deduce conditions of their use and compare with the results of

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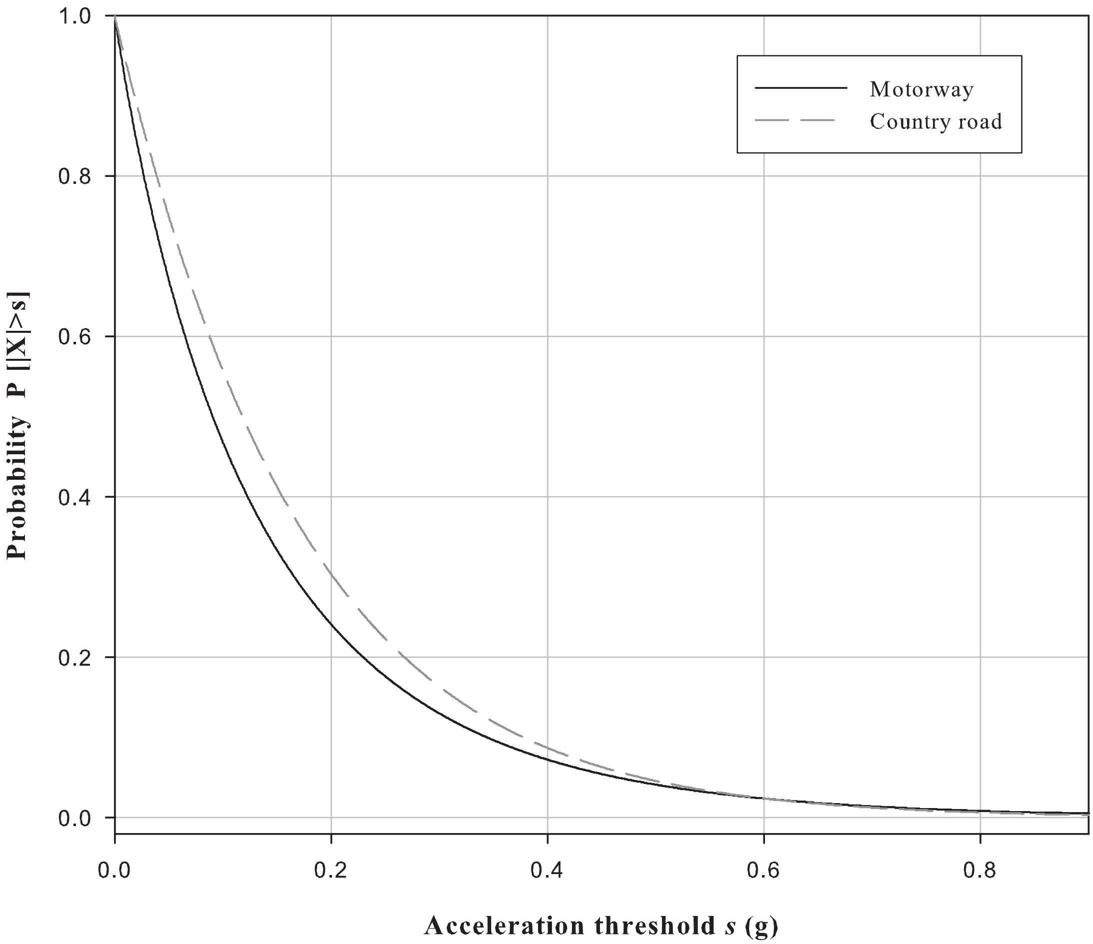

Figure 5. Appearance probability of acceleration levels greater than a fixed threshold as a function of this threshold.

Rouillard's work[5,14] in which he has studied the effects of sampling parameters on descriptors of random vibration processes, such as the average PSD and the RMS distribution.

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